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Equilibrium shapes and Geology of TNOS

a work in progress

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Abstract. Global fluid behaviour is usually assumed in the study of the equilibrium shapes, and therefore properties, of trans-neptunian objects (TNOs). However, recent results indicate that this simplification is not always valid. TNOs are composed by a mixture of ice and rock, the last generally in the shape of dust and/or grains of various sizes. The properties of these mixtures not only allow the existence of states with residual global stress, but also a gamma of equilibrium shapes different of those assumed by a fluid (i.e. Maclaurin and Jacobi ellipsoids). Therefore, it is necessary to use geological criteria to obtain better estimations of the physical properties of these bodies. This allows correlations between parameters associated with the resistance of the materials and the application of forces, such as the ones related to spin and gravitational interactions. When using the spin velocity and the tridimensional shape, it is possible to infer parameters such as density and internal angle of friction, amongst others, that can be used as base for assumptions regarding composition, internal layering and, in some cases, possible geological processes acting on the body such as cryovolcanism. In order to do that, the method proposed by Holsapple 2004 was applied for high precision data (obtained from stellar occultations) of the TNOs Haumea, Quaoar, 2003 VS2 and 2002 GZ32, and the asteroids Ceres, Lutetia and Vesta. For the TNOs, Pluto and Charon were used as analogs for assumptions regarding geological processes and surface evolution.

Resumo. Classicamente assume-se comportamento global fluido no estudo das formas de equilíbrio e propriedades físicas de objetos transnetunianos (TNOs). Contudo, resultados recentes apontam que tal simplificação nem sempre é válida. TNOs são compostos por misturas de gelo e rocha, no geral sob a forma de poeira ou grãos de tamanhos diversos, o que resulta em comportamento distinto de um fluido. As propriedades desses materiais permitem não só a existência de estados de equilíbrio com stress residual, como de uma gama de formas de equilíbrio distintas do caso fluido. Para se obter estimativas mais precisas acerca das propriedades físicas desses astros, tal como densidade, se faz necessário o uso de critérios geológicos, como proposto por Holsapple (2001, 2004 e 2007). O uso de tais métodos permite correlacionar parâmetros associados a resistência do material às forças aplicadas sobre os corpos. Ao se utilizar forma e velocidade de rotação, é possível estimar valores de densidade e de parâmetros vinculados a resistência interna dos materiais que compõe esses astros (i.e. ângulo de atrito interno), que por sua vez permitem inferências sobre o tipo de material que os compõe, ou que compõe suas camadas mais superficiais. O método de Holsapple 2004 foi aplicado para dados de alta precisão de alguns TNOs, incluindo Haumea, Quaoar, e 2003 VS2, e de asteróides como Ceres, Lutetia e Vesta. No caso dos TNOs, notou-se comportamento muito próximo a um fluido, permitindo inferir camadas externas compostas majoritariamente por gelo. No caso dos asteróides, notou-se considerável influência da resistência dos materiais em sua forma, corroborando as inferências de composição predominantemente silicática.

Keywords. TNOs, Equilibrium Shapes, Solar System

1. Introduction

The possible shapes and spin velocities of bodies in the Universe are a consequence of their composition and the balance of forces acting upon them (i.e. self-gravity, tidal and centripetal forces) (Holsapple 2004). When they are balanced, the body is considered to be in equilibrium shape.

Holsapple 2001, 2004, 2007 proposed the use of geological stability criteria to study the small bodies of the Solar System. The method is based on the evaluation of the limit-state of the bodies, in which the maximum internal tensions are accommodated without deformation or significant shape modification. It allows the inference of relationships between the tridimensional shape, angle of internal friction (ϕ), and spin velocity (ω). From these, it's possible to infer physical properties such as density (ρ) and maximum spin velocity supported by the body in its current shape.

The results presented here are still being refined. Until the present, the method was applied to high precision data obtained from stellar occultation by the TNOs Haumea, Quaoar and 2003 VS2, and the centaur 2002 GZ32, and to space probe data of Lutetia (obtained by Rosetta space-probe).

2. Methods

The method proposed by Holsapple 2004 was used to infer the physical properties (ρ and/or ϕ) of the small bodies listed above. It allows the determination of scaled spin (Ω) as a function of the aspect ratio α (c/a), where c is the shortest semi-axis of the object and a is the longest) and the ϕ for a certain shape (i.e. prolate, oblate or spheroidal).

The parameter Ω is calculated from the spin velocity ω and the density ρ using equation 1.

$$\Omega^2 = \frac{\omega^2}{\pi\rho G} \tag{1}$$

Curves of Ω vs. α were calculated, using literature values, for a range of ϕ (grey lines in the figures). Each curve represents all possible combinations of spin velocity and shape for a certain angle of internal friction. Their shape is a function not only of ϕ, but also of the shape of the body (aspect ratio β, or b/a, where b is the intermediate semi-axis of the object).

Object	α	β	ρ (kg/m^3)	Rotation Period (h)
Quaoar[1]	0.910 (+0.027/-0.017)	~1	1990 ± 460	8.8394 ± 0.0002
Haumea	0.442 (+0.016/-0.017) [2]	0.7339 [2]	1885 ± 80 [2]	3.915341 ± 0.000005 [3]
2003 VS2[4]	0.788 (+0.085/-0.139)	0.846 (+0.031/-0.028)	-	7.41753 ± 0.00001
2002 GZ32	0.4058 [3]	0.8841 [4]	1050 (+500/-350) [4]	5.80 ± 0.03 [5]
Lutetia	0.620 (+0.114/-0.112) [6]	0.835 ± 0.015 [6]	3400 ± 300 [8]	8.168270 ± 0.000001 [7]

Table 1. Values of α, β, ρ and period of rotation for the studied objects. References: [1]Braga-Ribas 2013, [2]Ortiz et al. 2017, [3]Santos-Sanz et al. 2018, [4]Benedetti-Rossi et al. 2019, [5]Dotto et al. 2008, [6]Carry et al. 2012, [7]Carry et al. 2010, [8]Patzoid et al. 2011.

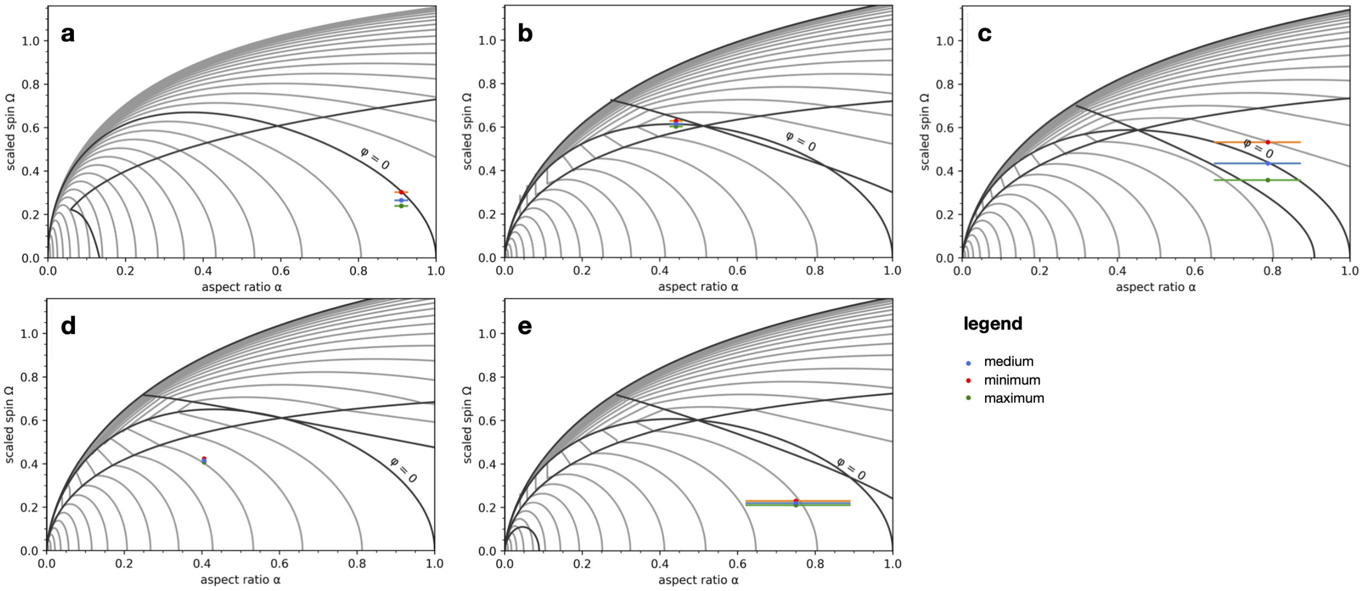

Figure 1. Plot of Ω vs. α, where each curve corresponds to a different angle of friction, spaced by 5^o, for a) Quaoar, b) Haumea, c) 2003 VS2, d) 2002 GZ32, and e) Lutetia. The coloured points and associated error bars are obtained calculating the values of α and Ω for the object, using the average (in blue), minimum (in red) and maximum (in green) values of density, as described in the text.

3. Results

The rotation period, semi-axis a, b and c (used to calculate the aspect ratios α and β), and ρ (when available) were taken from the available bibliography and are summarised in Tab. 1. They were used to calculate the curves for each object, according to its shape. The plots were used to obtain values of ϕ and ρ, used to make interpretations regarding composition, internal structure and, when possible, probable geological processes.

According to Fig. 1a, Quaoar have a behaviour very close to a fluid, assuming a Maclaurin equilibrium shape. Small deviations from fluid behaviour is possibly a consequence of the presence of rocky material. This verifies the validity of the method, due to the fact that the semi-axis values used here were obtained considering a purely fluid behaviour for the object.

Haumea's plot (Fig. 1b) indicates that the object is in equilibrium, but with different shape from a Jacobi ellipsoid. Its behaviour is similar to a fluid, possibly due to an upper layer composed mostly by ice. However, its density indicates the possibility that it has a denser (rocky) core, allowing the assumption that the object is differentiated and has a layered structure.

2003 VS2 was the only object without a value of ρ available in the bibliography. Its graph (Fig. 1c) was used to estimate its density, under the assumption that it has a fluid behaviour ($\phi = 0$). It is shown that the object is in equilibrium, with a shape different of a Jacobi ellipsoid, and density of 1050 $(+500/-350)$ kg/m^3.

2002 GZ32 is one of the 4 centaurs with multi-chord occultation data available (Santos-Sanz et al. 2018). Its plot (Fig. 1d) indicates that it is in equilibrium, but with shape different from a Jacobi ellipsoid, and $\phi = 15$ deg. This value is compatible with some geological materials found on Earth, such as sand and pebble aggregates, and can be an indicative that the object is composed by a mixture of rock and ice, with a considerable fraction of rocky material and possibly high porosity.

Lutetia's plot (Fig. 1e) indicates $\phi = 5°$, a value compatible with certain types of sediment aggregates found on Earth. It possibly have a rubble-pile structure, as is believed to be the case for most asteroids. It also validates the method, allowing comparisons with other methods.

4. Discussion and conclusions

It was noticed that the studied bodies are, generally speaking, in equilibrium, although with shapes different than the fluid cases (Maclaurin and Jacobi ellipsoids). The difference is usually quite small, what characterises studies considering fluid behaviours as reasonable first approximations.

For bodies with shapes well characterized through stellar occultations, that have shapes different from that expected for the fluid cases, this method allows a first estimate of its density.

It was verified that the properties of the materials considerably affect the way the bodies respond to forces, and, consequentially, their equilibrium shapes. Even small fractions of rocky material are sufficient to significantly change their shapes, as is the case for most TNOs. However, more detailed studies are required in order to reach further conclusions.

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